ANALYSIS OF RECONFIGURED CONTROL LOOP WITH A VIRTUAL ACTUATOR

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Abstract. *Control reconfiguration changes the control structure in response to a fault detected in the plant. This becomes necessary, because a major fault like loss of an actuator breaks the corresponding control loop and therefore renders the whole system inoperable. An important aim of control reconfiguration is to change the control structure as little as possible, since every change bears the potential of practical problems. The proposed solution is to keep the original controller in the loop and to add an extension called virtual actuator that implements the necessary changes of the control structure. The virtual actuator translates between the signals of the nominal controller and the signal of the faulty plants. This paper is concerned with the analysis of reconfigured loop with a virtual actuator for the system with the faulty actuator. The proposed analysis is illustrated on numerical example.*

Keywords

Fault tolerant control, systems, state control.

1. Introduction

All technological systems are subject to faults, due to both component malfunctions and unforeseen external influences. The complexity of control systems requires fault tolerance schemes to provide control of the faulty system. Fault tolerant systems are that one of the more fruitful applications with potential significance for domains in which control of systems must proceed while the system is operative and testing opportunities are limited by operational considerations. The real problem is usually to fix the system with faults so that it can continue its mission for some time with some limitations of functionality.

The main task to be tackled in achieving fault-tolerance is design of controller with such suitable reconfigurable structure which guarantees the stability, the satisfactory performance and the plant operation economy in nominal operational conditions. To achieve the fault tolerance used methods rely on employing on line fault diagnosis schemes which activate an alternative control – reconfigurable control structure- that is supposed to handle a fault.

Among these structures can be quoted control systems with adaptation to faults, the virtual-based control structure, as well as the output control reconfiguration algorithms [4].

In order to solve the complexity problems the control reconfiguration has to satisfy the requirement [3], [6] that the control reconfiguration has to be performed on line after the fault has been detected. This requires simple and fast algorithms that work reliably without manual interventions and without tuning the controller parameters for the fault case. It is sufficient to store a parametric model of the system (including all faults) and a reconfiguration algorithm. The new control structure is generated on demand after the fault has been detected. Bibliographical reviews can be found in [5], [8], [11], new developments in fault-tolerant control methods are presented e.g. in [1], [2], [6], [7], [9].

This paper is concerned with the problem analysis of reconfigured control loop with a virtual actuator for the system with the faulty actuator. Controller switching is taking into account since such different faulty system representations are known and stabilizing controllers are pre-computed off-line.

2. Problem Description

The presented task is concerned with the computation of the adaptive state feedback $\mathbf{u}(t)$, which control the faulty linear dynamic system.

An actuator fault is modelled by means of changing the input map $\mathbf{B}$ towards $\mathbf{B}_f$. Columns of $\mathbf{B}_f$ that correspond to faulty actuators are scaled in case of actuator degradation, or set to zero in the case of actuator failure. The task is concerned with the computation of the adaptive state feedback $\boldsymbol{u}(t)$, which control the faulty linear dynamic system given by the set of equations:

$$\dot{x}_f(\mathrm{t}) = Ax_f(\mathrm{t}) + B_f u_f(\mathrm{t}), \quad (1)$$

$$y_f(\mathrm{t}) = Cx_f(\mathrm{t}), \quad (2)$$

where $\mathbf{x_f}(\mathrm{t}) \in \mathrm{R}^n$, $\mathbf{u_f}(\mathrm{t}) \in \mathrm{R}^r$ and $\mathbf{y_f}(\mathrm{t}) \in \mathrm{R}^m$ are vectors of the state, input and output variables, respectively, and $\mathbf{A} \in \mathrm{R}^{n\times n}$, $\mathbf{B_f} \in \mathrm{R}^{n\times r}$, $\mathbf{C} \in \mathrm{R}^{m\times n}$ are real matrices of full ranks for the fault system.

If the nominal controller is a proportional state feedback controller:

$$u_c(\mathrm{t}) = K(w(\mathrm{t}) - y_c(\mathrm{t})) = K(w(\mathrm{t}) - Cx(\mathrm{t})), \quad (3)$$

the closed – loop model with nominal plant model is in the form:

$$\dot{x}(\mathrm{t}) = (A - BKC)x(\mathrm{t}) + BKw(\mathrm{t}), \quad (4)$$

where **K** is the nominal controller gain matrix.

The goal is to find a reconfiguration block, such that the state $\mathbf{x}_f(\mathrm{t})$ in the reconfigured control loop follows exactly the same trajectory as the state $\mathbf{x}(\mathrm{t})$ in the nominal loop.

3. Virtual Actuator

The stabilisation goal requires the reconfigured control loop to be stable. It is further required that the signals of the controller are not affected by the fault. Since the idea of the reconfiguration is to make the faulty plant behave like the nominal plant, the state of the model of the nominal plant is used as a reference. The control law is given

$$u_f(\mathrm{t}) = M(x(\mathrm{t}) - x_f(\mathrm{t})). \quad (5)$$

In context with [2] the structure of the reconfigured loop with a virtual actuator is on Fig. 1 and the virtual actuator is defined by the state–space model:

$$\dot{e}(\mathrm{t}) = (A - B_f M)e(\mathrm{t}) + Bu_c(\mathrm{t}), \quad (6)$$

$$y_c(\mathrm{t}) = y_f(\mathrm{t}) + Ce(\mathrm{t}), \quad (7)$$

$$u_f(\mathrm{t}) = Me(\mathrm{t}), \quad (8)$$

$$e(0) = 0, \quad (9)$$

where the matrix **M** is stabilising the pair (**A**, $\mathbf{B}_f$) such that the poles (**A** - $\mathbf{B}_f$**M**) are within the design set.

The state of the virtual actuator represents the deviation of the faulty plan state from its nominal value. The reconfigured plant can be constructed from the faulty plant and the virtual actuator, where the matrix of dynamic and input matrix are given in form:

$$\begin{bmatrix} A & 0 \\ 0 & A - B_f M \end{bmatrix}. \quad (10)$$

The two distinct subspaces are now obvious, since the transformed model consists of two subsystems without coupling.

This effect is known as separation principle; both parts can be designed separately. The poles of the system depend on the nominal controller be designed for the nominal system, while the poles of the virtual actuator depend on **M** as closed during the reconfiguration.

Several common design methods can be used to find such a matrices **M** and **K**, like pole placement or linear quadratic control design.

4. Controller and Virtual Actuator

4.1 Nominal System Controller Synthesis

Considering the system model parameter matrices **A, B, C** as follows:

$$A = \begin{bmatrix} 0{,}00 & 1{,}00 & 0{,}00 & 0{,}05 & 0{,}00 & 0{,}05 \\ 2{,}00 & -1{,}00 & 0{,}08 & 0{,}01 & 0{,}08 & 0{,}01 \\ 0{,}00 & 0{,}05 & 0{,}00 & -1{,}00 & 0{,}00 & 0{,}05 \\ 0{,}08 & 0{,}01 & -2{,}00 & 0{,}50 & 0{,}08 & 0{,}01 \\ 0{,}00 & 0{,}05 & 0{,}00 & 0{,}05 & 0{,}00 & 1{,}00 \\ 0{,}08 & 0{,}01 & 0{,}08 & 0{,}01 & 1{,}00 & -2{,}00 \end{bmatrix}, \quad (11)$$

$$B^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (12)$$

$$Cz = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad (13)$$

and using *Matlab* function **K** = *place*(A,B,r) to pole placement method design there was computed the controller gain matrix **K**. Note, ***r*** is the vector of the set desired eigenvalues of the closed-loop system matrix, i.e.

$$\sigma(A - BK) = \{-0{,}8 \quad -3 \quad -1.2 \quad -9 \quad -5 \quad -1\}, \quad (14)$$

and **K** was computed as

$$K = \begin{bmatrix} 10{,}07 & 7{,}23 & 0{,}56 & 0{,}06 & 3{,}47 & 2{,}96 \\ 0{,}39 & 0{,}06 & -6{,}97 & 6{,}50 & 0{,}36 & 0{,}04 \\ 3{,}72 & 2{,}97 & 0{,}44 & 0{,}05 & 5{,}74 & 3{,}78 \end{bmatrix}, \quad (15)$$

The simulation results are shown in Fig. 2 and Fig. 3.

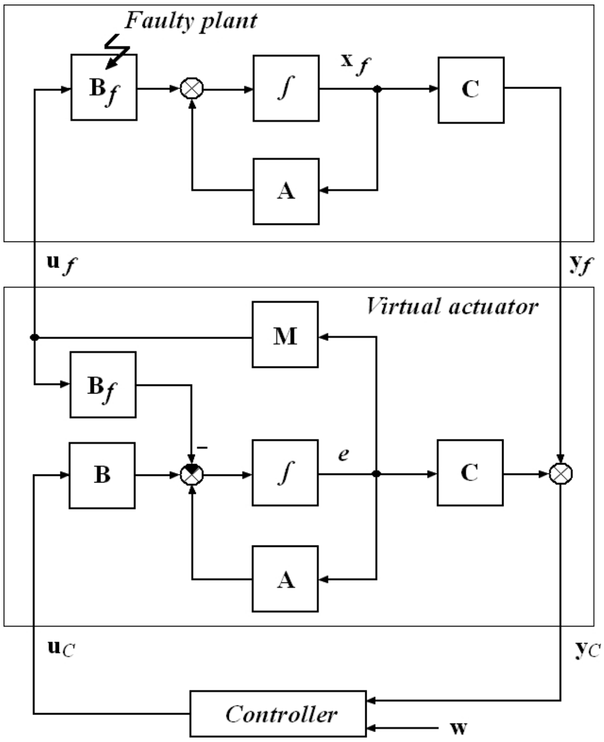

Fig. 1: Block diagram of the virtual actuator.

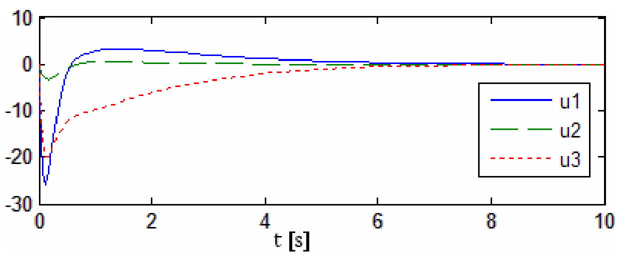

Fig. 2: Control variables $u1$, $u2$ and $u3$ of nominal system.

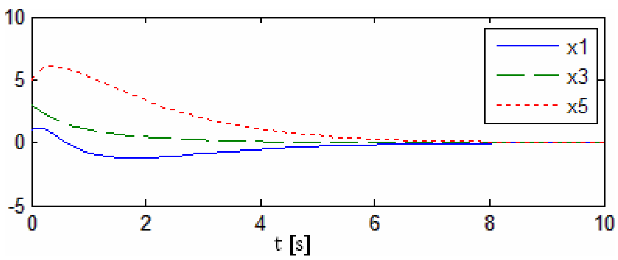

Fig. 3: State variables $x1$, $x3$ and $x5$ of nominal system.

4.2 Virtual Actuator Synthesis

At the loss of actuator $\boldsymbol{u}_1$ the input matrix **B** is affected by the fault, and takes the next form:

$$B^T{}_{f1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (16)$$

Therefore, the roots of the faulty closed loop character-ristic equation are:

$$\sigma(A - B_{f1}K) = \{0{,}96 \quad -4{,}75 \quad -5{,}01 \quad -1{,}96 \quad -1{,}02 \quad -1\},$$
$$i = 1{,}2{,}3$$

and lead to an unstable closed loop system.

For the synthesis the virtual actuator matrix parameters there was used the same method where the closed loop eigenvalues were chosen as:

$$\sigma(A - B_{fi}M_i) = \{-0{,}8 \quad -3 \quad -1{,}2 \quad -9 \quad -5 \quad -1\},$$
$$i = 1{,}2{,}3$$

and the matrix $\mathbf{M_1}$ was obtained in form:

$$M_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 49{,}22 & 22{,}50 & -4{,}78 & 6{,}59 & 0{,}75 & 0{,}68 \\ 550{,}15 & 255{,}12 & 25{,}01 & 0{,}95 & 10{,}19 & 10{,}91 \end{bmatrix}.$$

The effect of the virtual actuator action can be seen in Fig. 4. and Fig. 5.

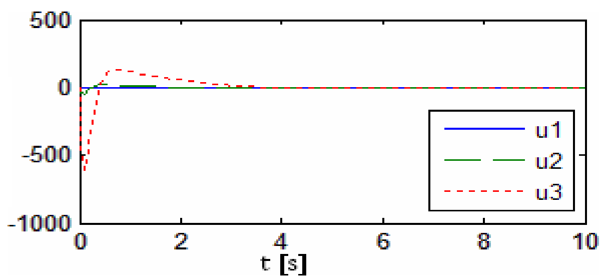

Fig. 4: Control variables $u1$, $u2$, $u3$ – the faulty system with virtual actuator – fault on $u1$.

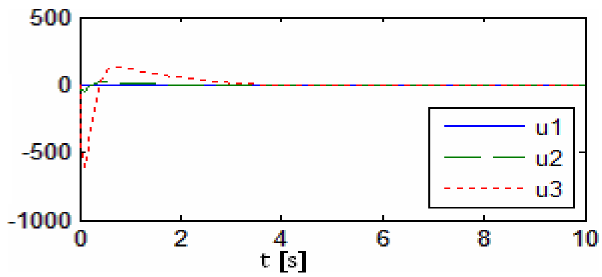

Fig. 5: State variables $x1$, $x3$, $x5$ of the faulty system with virtual actuator – fault on $u1$.

The next results implying from realized simulations showing in Fig. 2, Fig. 3, Fig. 4 and Fig. 5 can be generalized with respect to system time responses as follows: If an real actuator is loosed, and the virtual actuator is applied then

- the closed-loop system dynamic states stable,
- the loss of an actuator acts on all input and state variables of the system,
- the maximal values of the variables are increasing,

- time to fix the state variables is increasing.

5. System Responses

5.1 System Variables

Analyse of the closed-loop system with respect to the virtual actuator actions has to be done as for system input variables, as for system state variables.

The responses of above mentioned system variables in the structure without virtual actuator with respect to the loss of the first actuator realized 10 seconds after the state control start-up are shown in the Fig. 6 and Fig. 7.

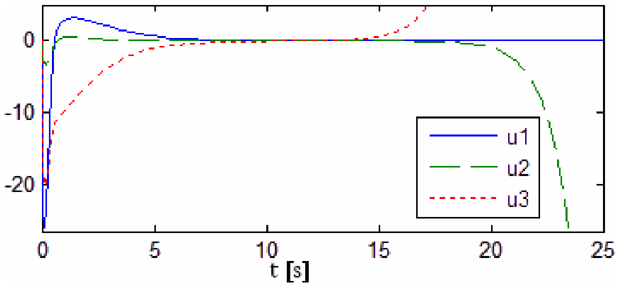

Fig. 6: Control variables $u1$, $u2$, $u3$, the fault $u1$ at 10 s.

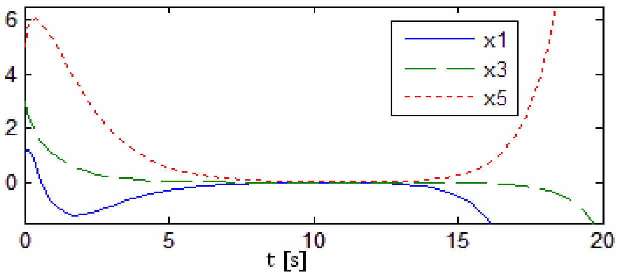

Fig. 7: State variables $x1$, $x3$, $x5$, the fault $u1$ at 10 s.

The responses of above mentioned system variables in the structure with virtual actuator with respect to the loss of the first actuator realized 10 seconds after the state control start-up and in dependence the time-delay in starting of virtual actuator are shown in Fig. 8, Fig. 9, Fig. 10 and Fig. 11. Here the time-delay starting time of virtual actuator was 3 and 5 seconds, respectively.

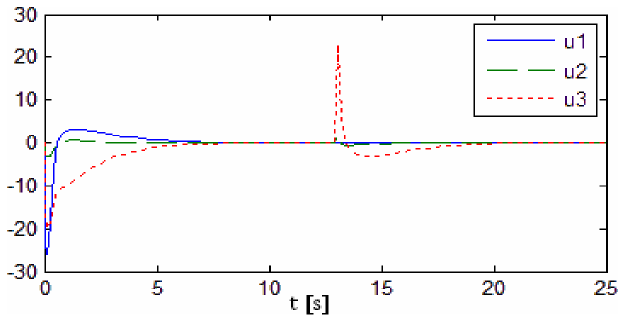

Fig. 8: Control variables $u1$, $u2$, $u3$, the fault starting time 10 s, the virtual actuator starting time 13 s.

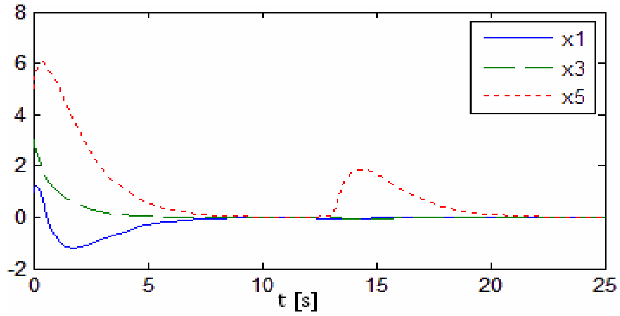

Fig. 9: State variables $x1$, $x3$, $x5$, the fault starting time 10 s, the virtual actuator starting time 13 s.

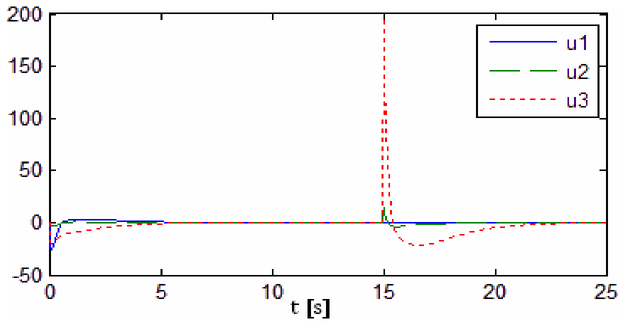

Fig. 10: Control variables $u1$, $u2$, $u3$, the fault starting time 10s, the virtual actuator starting time 15 s.

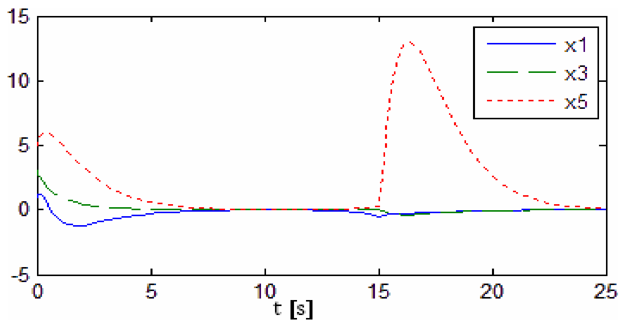

Fig. 11: State variables $x1$, $x3$, $x5$, the fault starting time 10s, the virtual actuator starting time 15 s.

Properties implying from the system variable time responses can be formulate as:

- There exists expressive influence of a time-delay in starting time of the virtual actuator on control system variables.
- Increasing the time-delay in starting time of virtual actuator results in expensive rising of the maximal value of the control variables, and increase claims on the operating fault-free actuators.

It is obvious that this implies the hard claims on the fault detection and isolation subsystem time responses.

5.2 Subsystem Interactions

Some influence of interactions among subsystems on system input and system state variables at the loss of an actuator and with the virtual actuator action present Tab. 1, 2 and 3, where the input and state variables of the system reflect the loss of the actuators at 10s and the virtual actuator activity start at 15s after the state control start-up.

The interactions among subsystems were increased 5, 10 and 19 times and the behaviour of the reconfigured control loop with a virtual actuator was followed through the influence on the maximum values of the control and state variables.

Tab.1: Fault of the actuating No.1.

	Maximal values of input and output system variables					
	|u1|	**|u2|**	**|u3|**	**|x1|**	**|x3|**	**|x5|**
1x	0	17,41	196,18	0,52	0,38	13,03
5x	0	0,55	5,01	0,09	0,02	0,43
10x	0	0,11	0,87	0,04	0,01	0,1
19x	0	10,39	79,32	2,4	2,62	2,14

Tab.2: Fault of the actuating No.2.

	Maximal values of input and output system variables					
	|u1|	**|u2|**	**|u3|**	**|x1|**	**|x3|**	**|x5|**
1x	17474,75	0	18394,1	910,05	82,8	950,76
5x	254,68	0	273,43	16,65	6,97	17,42
10x	477,8	0	516,7	41,01	32,4	42,32
19x	43014,5	0	21266,8	3976,8	5570	3772,9

Tab.3: Fault of the actuating No.3.

	Maximal values of input and output system variables					
	|u1|	**|u2|**	**|u3|**	**|x1|**	**|x3|**	**|x5|**
1x	12,04	6,31	0	0,88	0,33	0,07
5x	0,23	0,01	0	0,02	0,004	0,01
10x	0,7	0,06	0	0,07	0,02	0,05
19x	19,9	5,34	0	1,36	1,59	1,2

Note, it is interesting that the interactions among subsystems directly affect the maximum values of the control and state system variables if virtual actuators are used in the control loop reconfiguration.

From this it follows:

- There exists expressive influence of subsystems interactions on the control and state system variables when reconfigurable control structure includes a virtual actuator.
- If subsystems interactions are 5 and 10-times increased then it is expressive decreased claims to the operating actuators.

On account of that for the safety systems follow, e.g. the claims on the mechanical systems construction, light materials, etc.

6. Conclusion

It follows from the analysis that at the loss of an actuator and by sequent application of the virtual actuator, the dynamic system stays stable. The loss of an actuator acts on all input and state variables of the system, the maximal values of the system variables are increasing, the time of reaching the steady-state state variables is substantially increasing.

There exists expressive influence of the time-delay in virtual actuators start-up on control system variables. The increase of this time-delay expensive increase the maximal value of the control variables of the system and increase claims to the operating actuators. However, it implies the hard claims on the fault detection and isolation subsystem time responses.

It is interesting that raising values of subsystems interactions expressive decrease claims to the operating fault-free actuators, and makes more acceptable the time response of the output and state variables if a virtual actuator has to be included into the control structure.

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